1. Verify that $y = C_1e^x + C_2xe^x + C_3x^2e^x$ is a solution of $y''' - 3y'' + 3y' - y = 0$. (Here C_1, C_2, C_3 are constants.)

Solve the following ODEs.

2. $y' = e^x + \sin x - x^3$
3. $y' = (x-3)^3,$ $\quad y(0) = 1$
4. $y' = xe^{x^2},$ $\quad y(1) = e$
5. $y' = \dfrac{\sin x}{2 + \cos x},$ $\quad y(\pi) = 3$
6. $y' = \ln x,$ $\quad y(1) = 3$
7. $y' = e^x\sqrt{e^x + 3},$ $\quad y(0) = 1$
8. $y'' = e^{2x},$ $\quad y(0) = 0,\ y'(0) = e$

9. Verify the following IVP

$$y' = e^y + \frac{\sin y}{\cos x}, \qquad y(1) = 0$$

has an unique solution on some interval including 1.

10. Test whether each of the following differential equations is a linear first-order ODE or not.

(1). $y' + \sin(xy) = x^2$

(2). $y' - (\sin^2 x)y = x^2$

(3). $(x^2+1)y' - \dfrac{e^x}{x^2+3}y^2 = 0$

(4). $yy' - x^3 = 0$

(5). $(y')^2 - x^2y = 0$

11. State an interval where the solution exists.

(1). $y' - (\tan x)y = \cot x,$ $\quad y(\frac{\pi}{4}) = 0$

(2). $y' + xy + e^x = 0,$ $\quad y(105) = 1$

(3). $\ln(x^2 - 1)y' + y = e^{x^2},$ $\quad y(\pi) = \pi^2$

(4). $y' - \ln(x^2 + 1)y = \frac{1}{x+1},$ $\quad y(0) = -5$

(5). $y' + \frac{y}{x(x-1)(x+5)} = \frac{1}{(x+2)(x-3)},$ $\quad y(-1) = 3$